① A pebble is dropped into a calm pond casting ripples. The radius, $r$, of the outer ripple is increasing at a constant rate of 1 foot per second. When the radius is 4 ft. at what rate is the area, $A$, of the disturbed water changing?

② Air is being pumped into a spherical ballon at a rate of $4.5\ ft^3/min$. Find the rate of chage of the radius when $r = 2$ ft.

③ Use linearizations to approximate $\sqrt[3]{26}$. How close is your approximation?

① $A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2 \cdot \pi \cdot 4 \cdot 1$

$\frac{dA}{dt} = 8\pi \ ft^2/sec$

$\approx \underline{25.13}$

$A = \pi (4)^2$

$A = 16\pi \approx 50.265$

$A = \pi (4.1)^2 \approx 52.81$

2.54 ft 0.1 sec

$\underline{25.4}$ ft 1 sec

② $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = \cancel{3} \cdot \frac{4}{\cancel{3}} \pi r^2 \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$4.5 = 4 \cdot \pi \cdot 2^2 \frac{dr}{dt}$

$4.5 = 16\pi \frac{dr}{dt}$

4.5/(16 π)

$\frac{4.5}{(16\pi)} = \frac{dr}{dt}$

$\approx 0.09$ ft/min

② $V = \frac{4}{3}\pi r^3$

$$\frac{dV}{dt} = \cancel{3} \cdot \frac{4}{\cancel{3}}\pi r^2 \frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$4.5 = 4 \cdot \pi \cdot 2^2 \frac{dr}{dt}$$

$$4.5 = 16\pi \frac{dr}{dt}$$

4.5/(16 π)

$$\frac{4.5}{(16\pi)} = \frac{dr}{dt}$$

$\approx 0.09$ ft/min

pc

$V = \frac{4}{3}\pi(2)^3 \approx 33.51$

1 min later 4.5 more volume

$\approx 38.01 = \frac{4}{3}\pi r^3$

$$\sqrt[3]{\frac{38.01}{(4/3\,\pi)}} = r$$

$r \approx 2.086$

$2.086 - 2 = 0.086$

$L(x) = f(a) + f'(a)(x-a)$ of $\sqrt[3]{26}$

$f(a) = \sqrt[3]{x}$

$f'(a) = \dfrac{1}{3(\sqrt[3]{x})^2}$

$\sqrt[3]{27} + \dfrac{1}{3(\sqrt[3]{27})^2}(26-27)$

$a = 27$

$x = 26$

$2\frac{26}{27}$

$3 + \dfrac{1}{27}(-1) \approx 2.96296296$

$\sqrt[3]{26} \approx 2.962496068$

error $\approx 0.0005$ within $10^{-3}$

# Weekly Review 6

1) Let $f$ be a continuous, differentiable, and monotonic function on the domain [3, 8]. The table shows four function values of $f$.

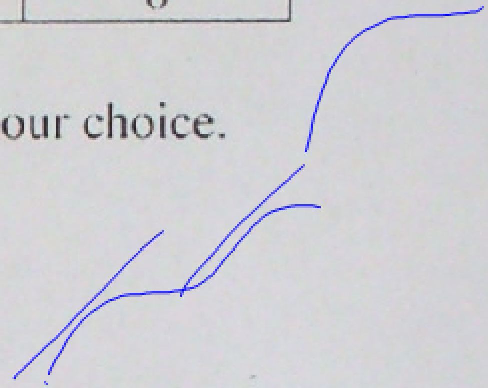
| $x$ | 3 | 4 | 6 | 7 |
|---|---|---|---|---|
| $f(x)$ | -4 | 1 | 5 | 8 |

Which of the following statements must be true? Explain your choice.

I. $f(8) > 9$ X

II. $f'(5) \geq 0$ True $f'(5) > 0$ X

III. $f'(c) = 3$ for exactly one $c$ in [3, 7] X



(A) II only

(B) II and III only

(C) III only

~~(D) I and III only~~

~~(E) I, II, III~~

(F) None of these

2) Let $g$ be a function defined and continuous on the closed interval $[a, b]$. If $g$ has a local minimum at $c$ where $a < c < b$, which of the following statements must be true? Explain your choice.

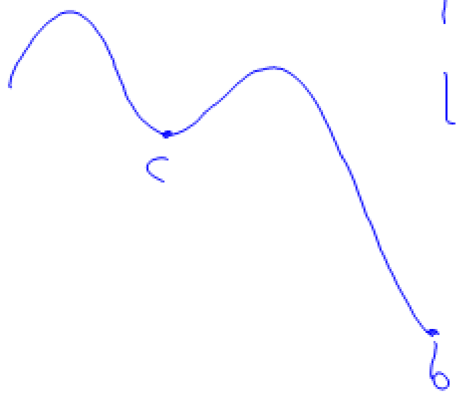
I. If $g'(c)$ exists, then $g'(c) = 0$ True

II. $g(c) < g(b)$ X

III. $g$ is monotonic on $[a, b]$ X

(A) I only (B) ~~II only~~ (C) ~~III only~~

(D) ~~I and II only~~ (E) ~~I and III only~~ (F) ~~None of these~~




1pt - Answer

1pt - Explanation

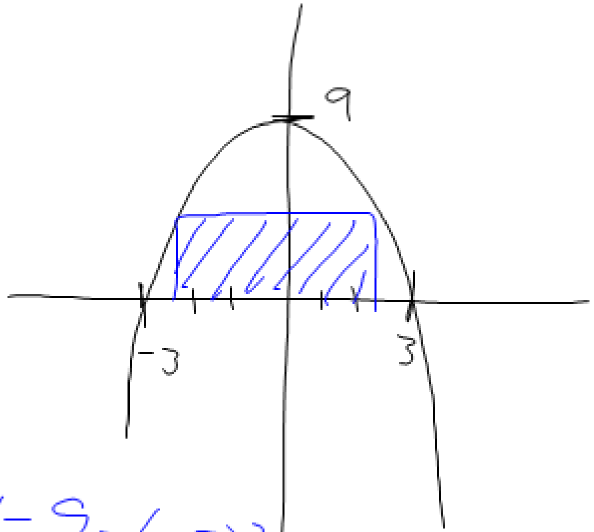
Problems #3 – 5 can be solved using algebra alone, but I want you to solve them using calculus.

3) A rectangle is inscribed under the graph of $h(x) = 9 - x^2$ and above the x-axis. Find the maximum possible area for that rectangle.

4) An open-topped box with a square base must be constructed with a volume of 12 cubic inches. What dimensions use the least amount of material? (*Hint:* Define your variables for dimensions of the box. Now write two equations, one for volume and one for surface area. Which one will be optimized?)

5) From an 8 inch by 10 inch rectangular sheet of paper, square of equal size will be cut from each corner. The flaps will then be folded up to form an open-topped box. Find the maximum possible volume of the box.

3) A rectangle is inscribed under the graph of $h(x) = 9 - x^2$ and above the x-axis. Find the maximum possible area for that rectangle.




$A = 2xy$

$y = 9 - x^2$

$A = 2x(9 - x^2)$

$A = -2x^3 + 18x$

$$\frac{dA}{dx} = -6x^2 + 18$$

$0 = -6x^2 + 18$

$x^2 = 3$

1pt. $\boxed{x = \pm\sqrt{3}}$

$-$ $\sqrt{3}$ $+$

max

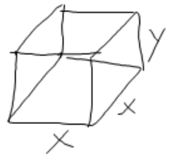
$y = 9 - (\pm\sqrt{3})^2$
$= 6$

$A = 2(\sqrt{3})(6)$

$\boxed{A = 12\sqrt{3} \text{ units}^2}$ 1pt.

4) An open-topped box with a square base must be constructed with a volume of 12 cubic inches. What dimensions use the least amount of material? (*Hint:* Define your variables for dimensions of the box. Now write two equations, one for volume and one for surface area. Which one will be optimized?)

$V = 12 \text{ in}^3$




$SA = x^2 + 4xy$

$SA = x^2 + 4x\left(\frac{12}{x^2}\right) \longrightarrow x^2 + \frac{48}{x}$

$V = x^2 y$

$12 = x^2 y$

$y = \frac{12}{x^2}$

$\frac{ds}{dx} = 2x + -\frac{48}{x^2}$

$0 = 2x - \frac{48}{x^2}$

$\frac{48}{x^2} = 2x$

$48 = 2x^3$

$24 = x^3$

$x = \sqrt[3]{24} \rightarrow \sqrt[3]{8} \cdot \sqrt[3]{3}$

$\boxed{x = 2\sqrt[3]{3} \approx 2.885}$

1pt.

$y = \frac{12}{(2\sqrt[3]{3})^2}$

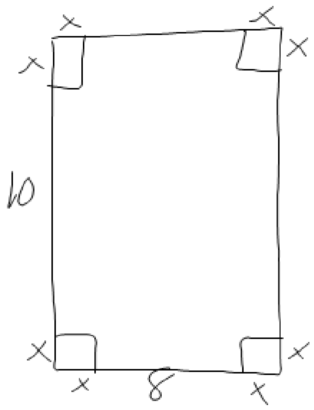
$y = \frac{12}{4\sqrt[3]{3^2}}$

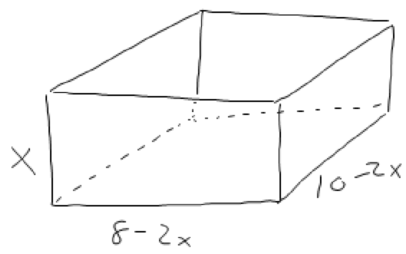
$\boxed{y = \frac{3}{\sqrt[3]{3^2}} \approx 1.44 \text{ in}}$

$y = \sqrt[3]{3} \text{ in}$

1pt.

5) From an 8 inch by 10 inch rectangular sheet of paper, square of equal size will be cut from each corner. The flaps will then be folded up to form an open-topped box. Find the maximum possible volume of the box.




$V = (x)(8-2x)(10-2x)$

$V = x(4x^2 - 36x + 80)$

$V = 4x^3 - 36x^2 + 80x$

$\frac{dV}{dx} = 12x^2 - 72x + 80$

Quad. formula

$x = 1.47, \cancel{4.53}$

1pt

$x = 1.47 \text{ in}$

$V(1.47) \approx 52.51 \text{ in}^3$

1pt.

Test on Thur over 4.4 – 4.6

$\boxed{4.4}$

#21-23, 27, 31, 33, 41

AP Stats - here

Calc 2 - FRCC
CU